Steady affine motions and morphs

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We propose to measure the quality of an affine motion by its steadiness, which we formulate as the inverse of its Average Relative Acceleration (ARA). Steady affine motions, for which \( \text{ARA}=0 \), include translations, rotations, screws, and the golden spiral. To facilitate the design of pleasing in-betweening motions that interpolate between an initial and a final pose (affine transformation), \( B \) and \( C \), we propose the Steady Affine Morph (SAM), defined as \( A^t ? B \) with \( A=C ? B^{-1} \). A SAM is affine-invariant and reversible. It preserves isometries (i.e., rigidity), similarities, and volume. Its velocity field is stationary both in the global and the local (moving) frames. Given a copy count, \( n \), the series of uniformly sampled poses, \( A^{(i/n)} ? B \), of a SAM form a regular pattern which may be easily controlled by changing \( B \), \( C \), or \( n \), and where consecutive poses are related by the same affinity \( A^{(1/n)} \). Although a real matrix \( A^t \) does not always exist, we show that it does for a convex and large subset of orientation-preserving affinities \( A \). Our fast and accurate Extraction of Affinity Roots (EAR) algorithm computes \( A^t \), when it exists, using closed-form expressions in two or in three dimensions. We discuss SAM applications to pattern design and animation and to key-frame interpolation.

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