Cage-based space deformations are often used to edit and animate images and geometric models. The deformations of the cage are easily transferred to the model by recomputing fixed convex combinations of the vertices of the cage, the control points. In current cage-based schemes the configuration of edges and facets between these control points affects the resulting deformations. In this paper we present a family of similar schemes that includes some of the current techniques, but also new schemes that depend only on the positions of the control points. We prove that these methods afford a solution under fairly general conditions and result in an easy and flexible way to deform objects using freely placed control points, with the necessary conditions of positivity and continuity.