Steady affine motions and morphs

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We propose to measure the quality of an affine motion by its steadiness, which we formulate as the inverse of its Average Relative Acceleration (ARA). Steady affine motions, for which ARA=0, include translations, rotations, screws, and the golden spiral. To facilitate the design of pleasing in-betweening motions that interpolate between an initial and a final pose (affine transformation), B and C, we propose the Steady Affine Morph (SAM), defined as A^t? B with A=C ? B^{?1}. A SAM is affine-invariant and reversible. It preserves isometries (i.e., rigidity), similarities, and volume. Its velocity field is stationary both in the global and the local (moving) frames. Given a copy count, n, the series of uniformly sampled poses, A^(i/n)? B, of a SAM form a regular pattern which may be easily controlled by changing B, C, or n, and where consecutive poses are related by the same affinity A^(1/n). Although a real matrix A^t does not always exist, we show that it does for a convex and large subset of orientation-preserving affinities A. Our fast and accurate Extraction of Affinity Roots (EAR) algorithm computes A^t, when it exists, using closed-form expressions in two or in three dimensions. We discuss SAM applications to pattern design and animation and to key-frame interpolation.

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